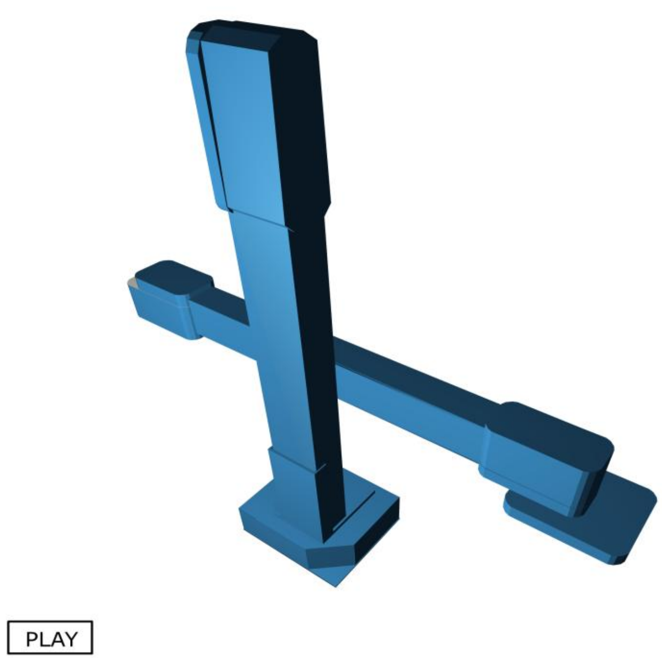
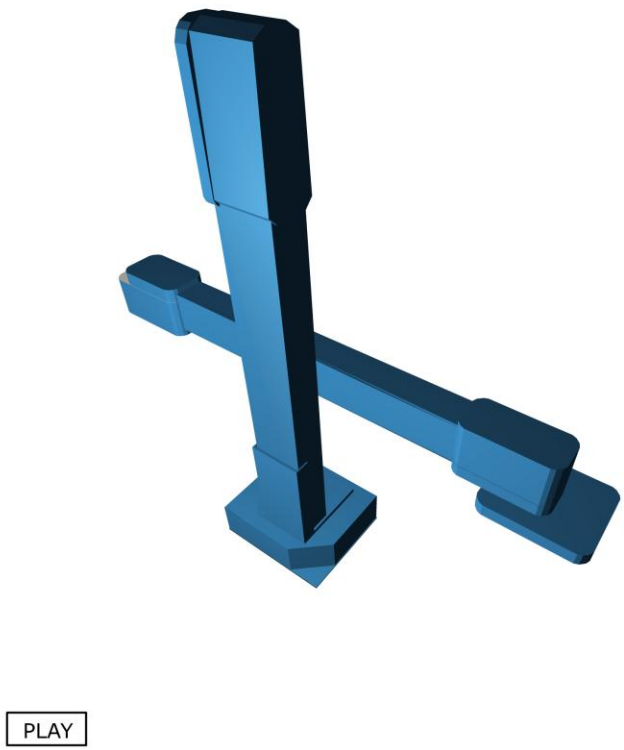
Day 08

Denavit-Hartenberg

Denavit-Hartenberg Forward Kinematics

- RPP cylindrical manipulator
  - http://strobotics.com/cylindrical-format-robot.htm
  - http://strobotics.com/robodem.htm





Denavit-Hartenberg Forward Kinematics

How do we place the frames?

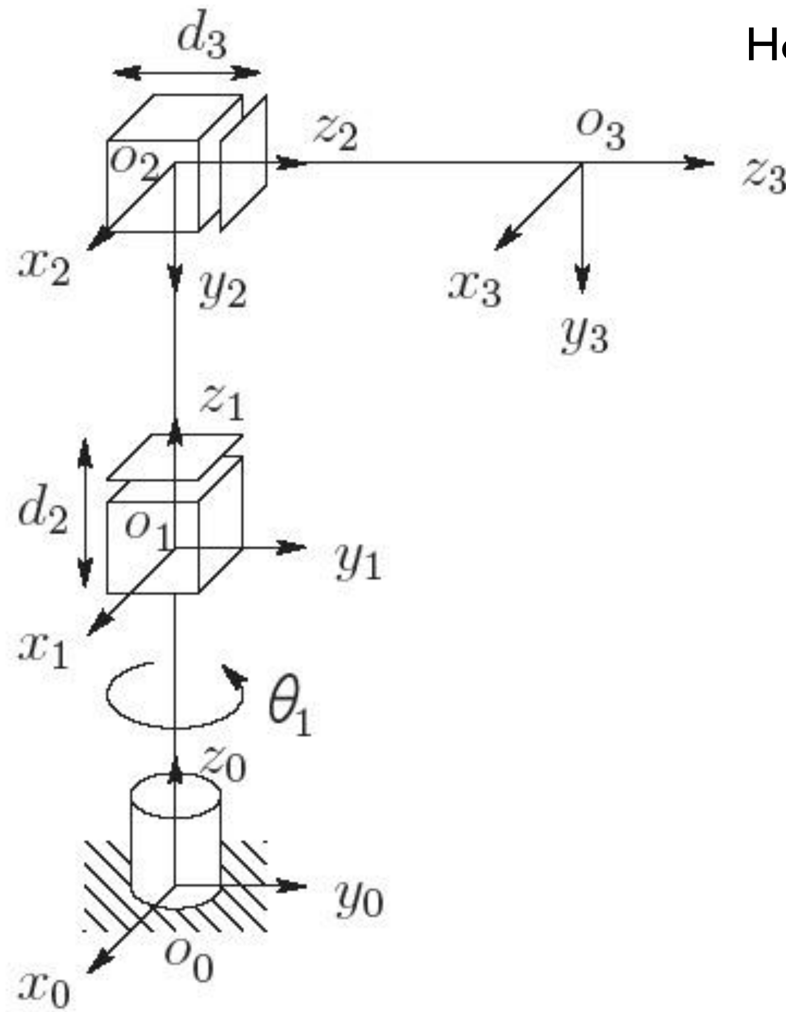


Figure 3.7: Three-link cylindrical manipulator.

Step 1: Choose the z-axis for each frame

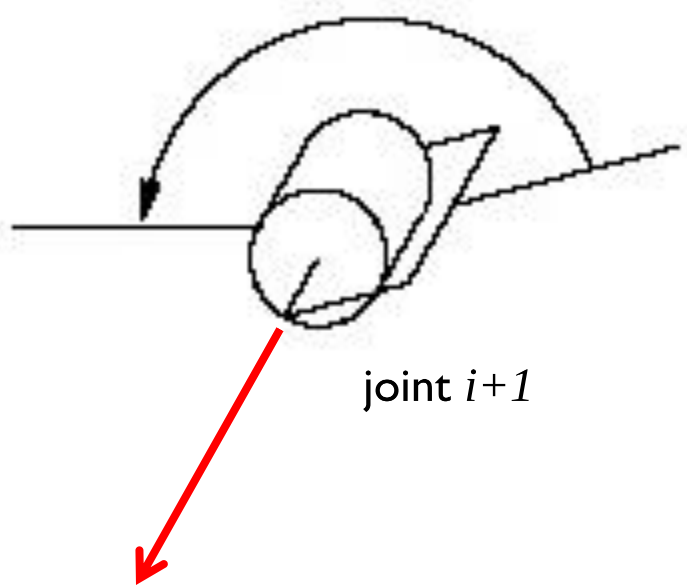
- recall the DH transformation matrix

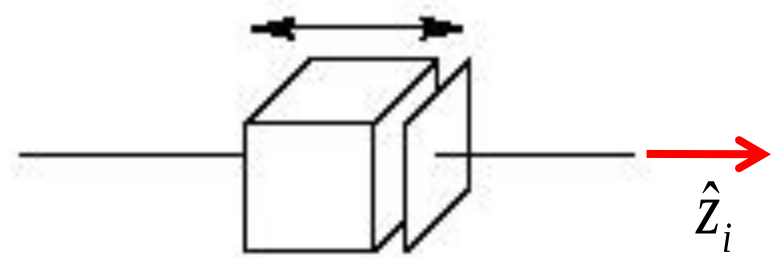
$$T_i^{i-1} = R_{z,\theta_i} T_{z,d_i} T_{x,a_i} R_{x,\alpha_i}$$

$$= \begin{array}{c} \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{cccc} \hat{x}_i^{i-1} & \hat{y}_i^{i-1} & \hat{z}_i^{i-1} & \phantom{a_i c_{\theta_i}} \end{array} \end{array}$$

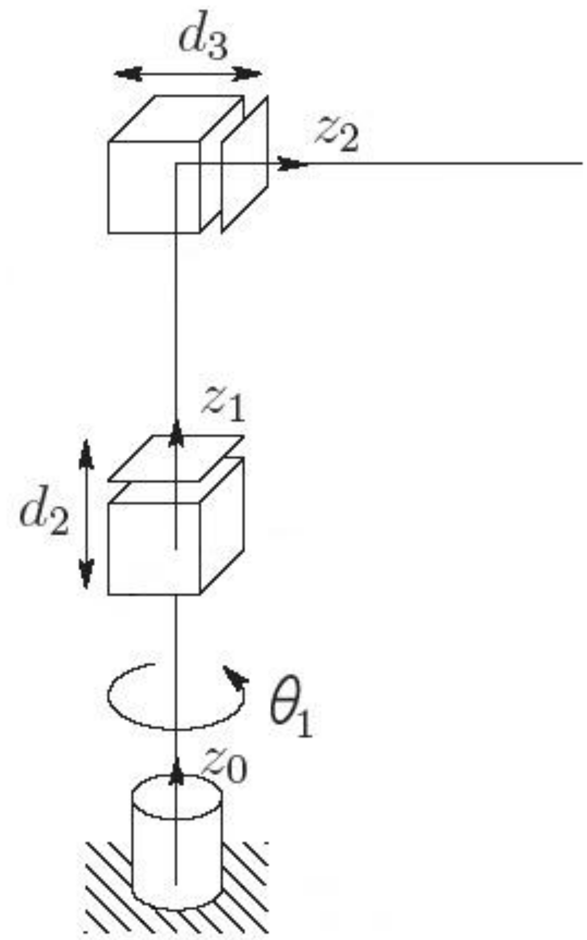
Step 1: Choose the z-axis for each frame

- $\hat{z}_i \equiv$ axis of actuation for joint $i+1$





Step 1: Choose the z-axis for each frame

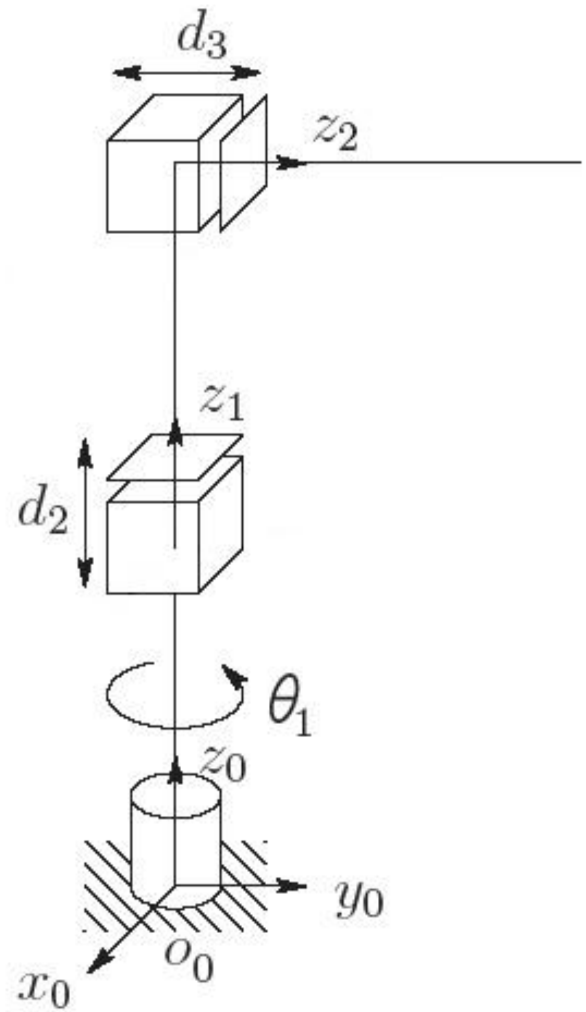


- Warning: the picture is deceiving. We do not yet know the origin of the frames; all we know at this point is that each z_i points along a joint axis

Step 2: Establish frame {0}

- place the origin o_0 anywhere on z_0
  - often the choice of location is obvious
- choose x_0 and y_0 so that $\{0\}$ is right-handed
  - often the choice of directions is obvious

Step 2: Establish frame {0}

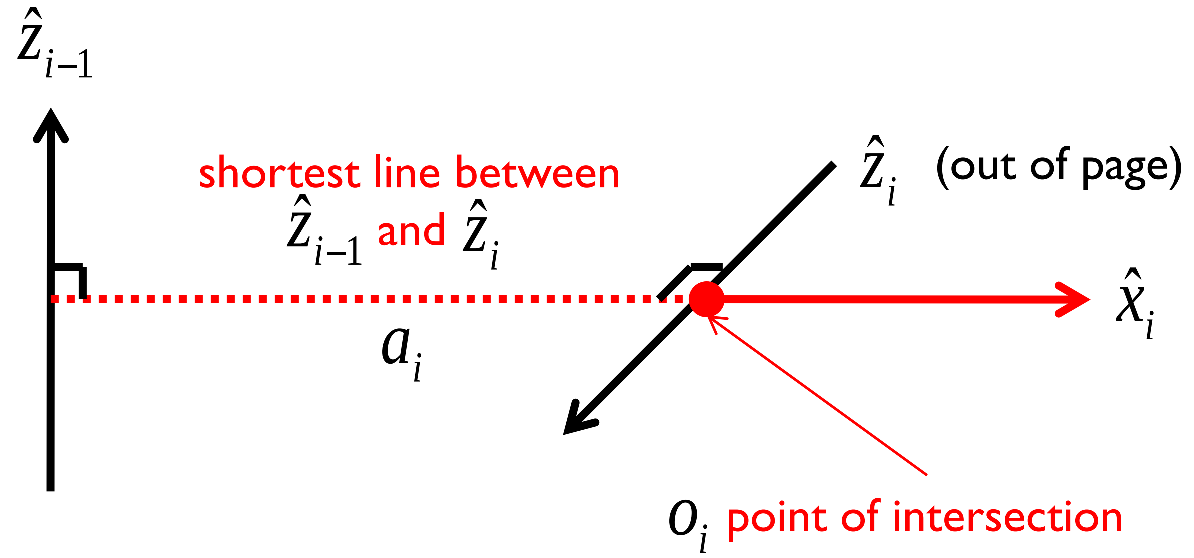


Step 3: Iteratively construct {1}, {2}, ... {n-1}

- using frame {i-1} construct frame {i}
  - DH1: x_i is perpendicular to z_{i-1}
  - DH2: x_i intersects z_{i-1}
- 3 cases to consider depending on the relationship between z_{i-1} and z_i

Step 3: Iteratively construct {1}, {2}, … {n-1}

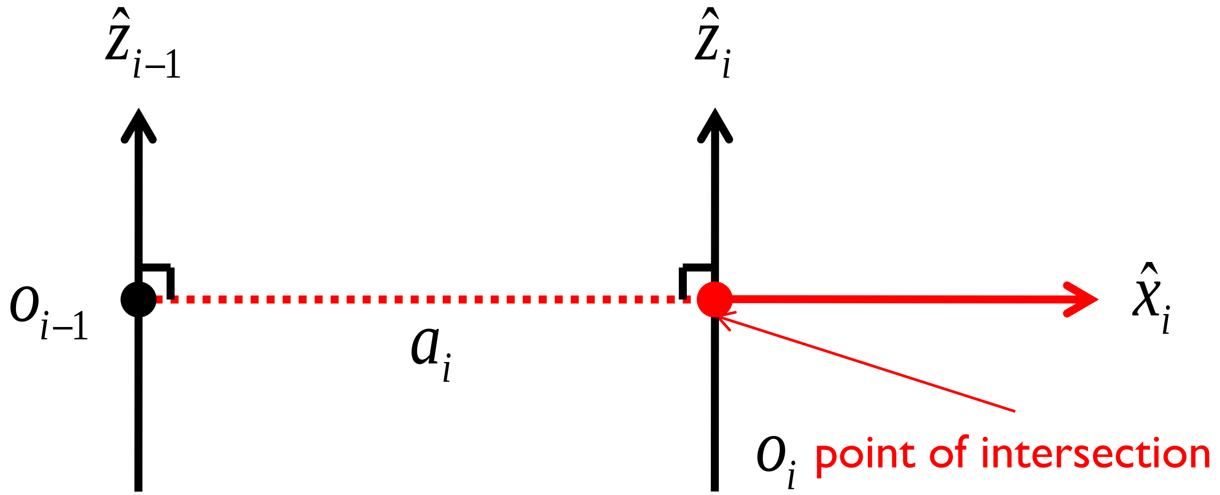
- Case 1
  - z_{i-1} and z_i are not coplanar (skew)



  - α_i angle from z_{i-1} to z_i measured about x_i

Step 3: Iteratively construct {1}, {2}, … {n-1}

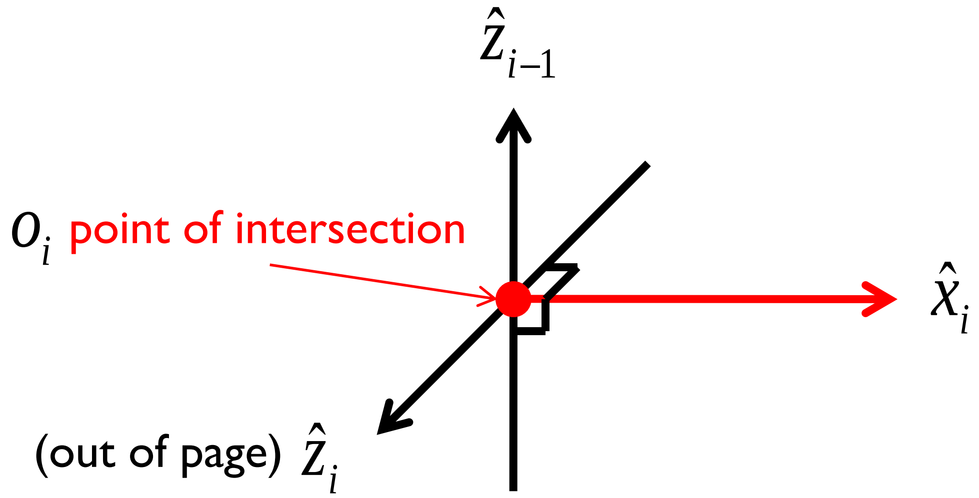
- Case 2
  - z_{i-1} and z_i are parallel ($\alpha_i = 0$)



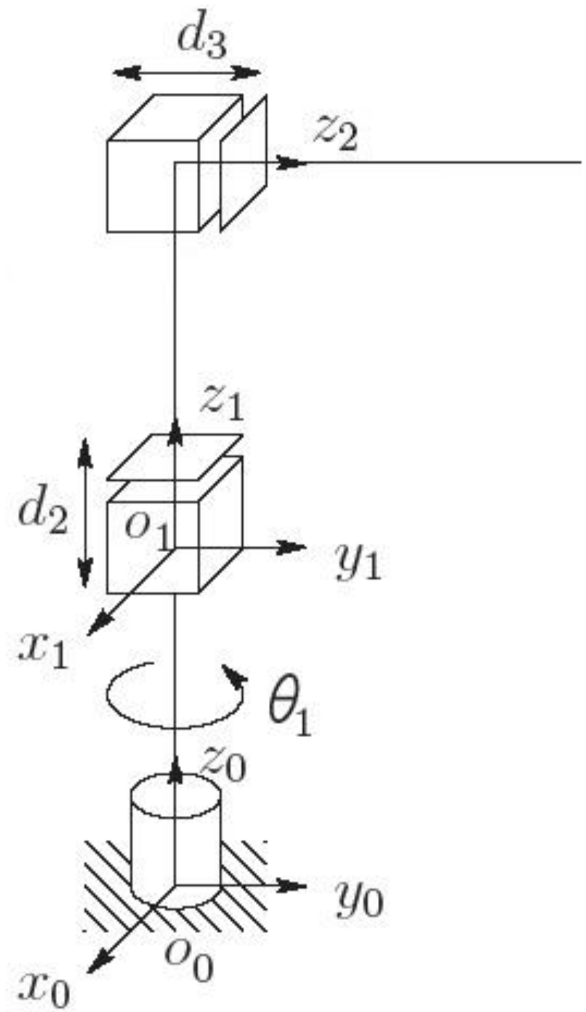
  - notice that this choice results in $d_i = 0$

Step 3: Iteratively construct {1}, {2}, … {n-1}

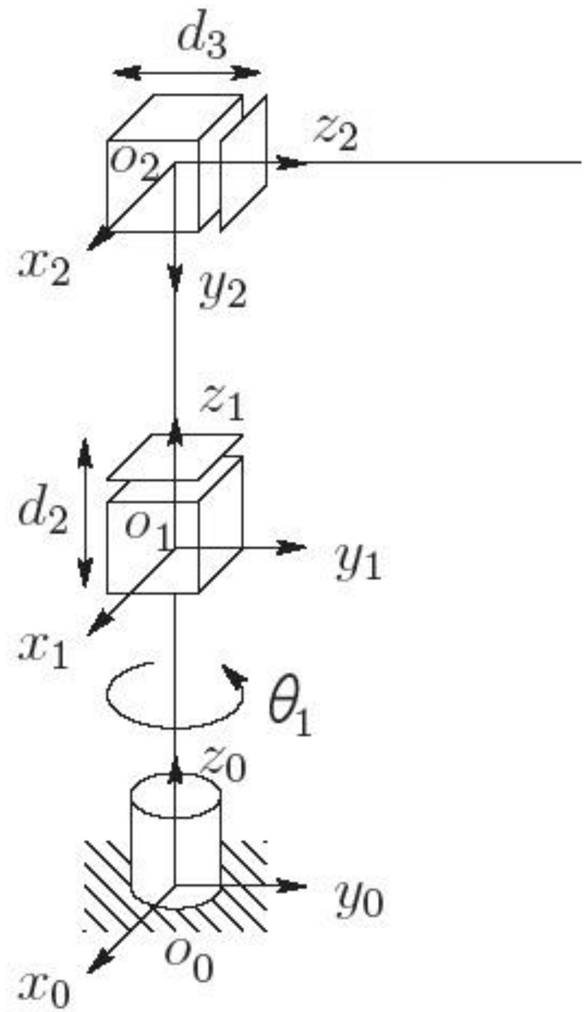
- Case 3
  - z_{i-1} and z_i intersect ($a_i = 0$)



Step 3: Iteratively construct {1}, {2}, … {*n*-1}



Step 3: Iteratively construct {1}, {2}, … {*n*-1}



Step 4: Place the end effector frame

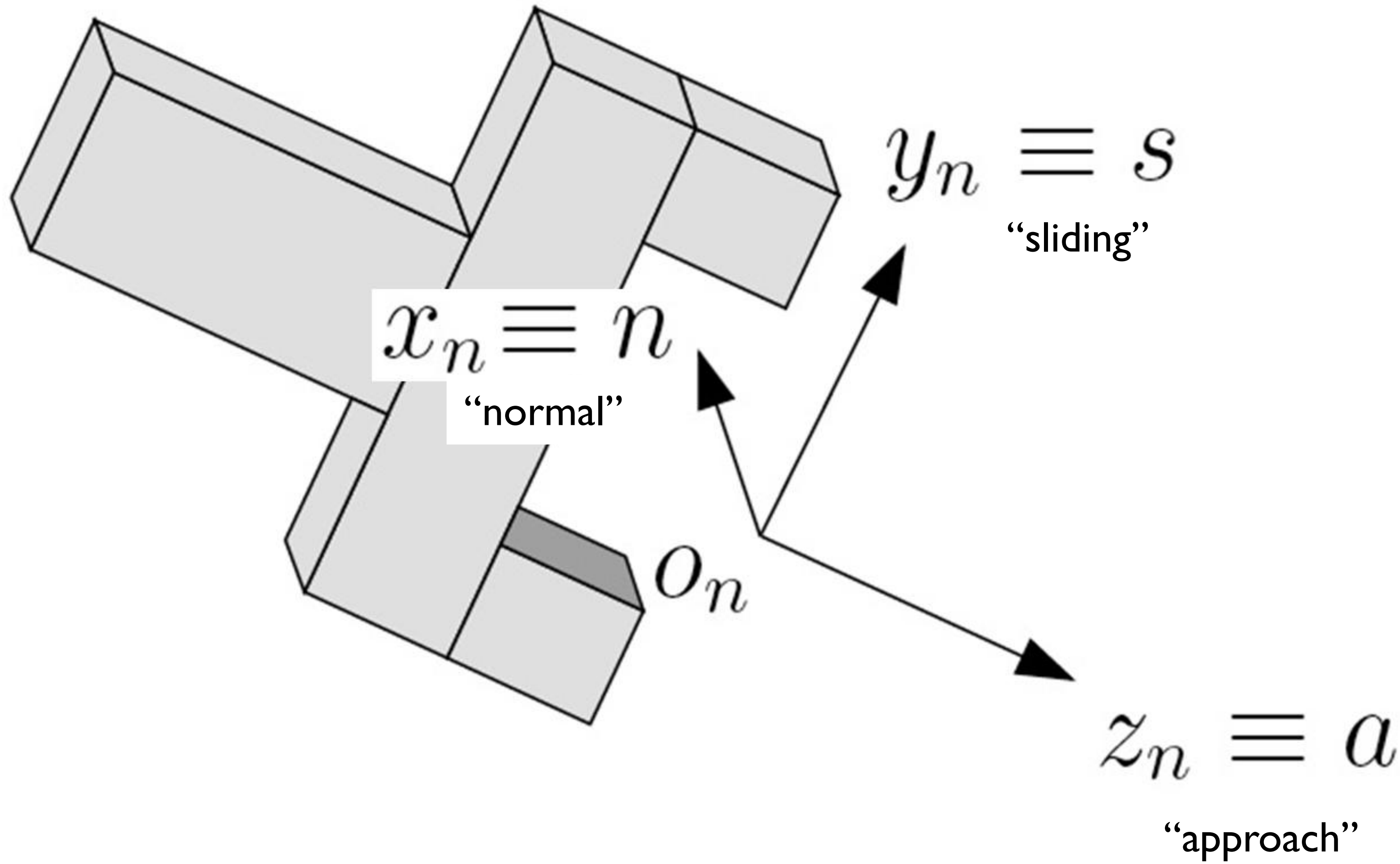


Figure 3.5: Tool frame assignment.

Step 4: Place the end effector frame

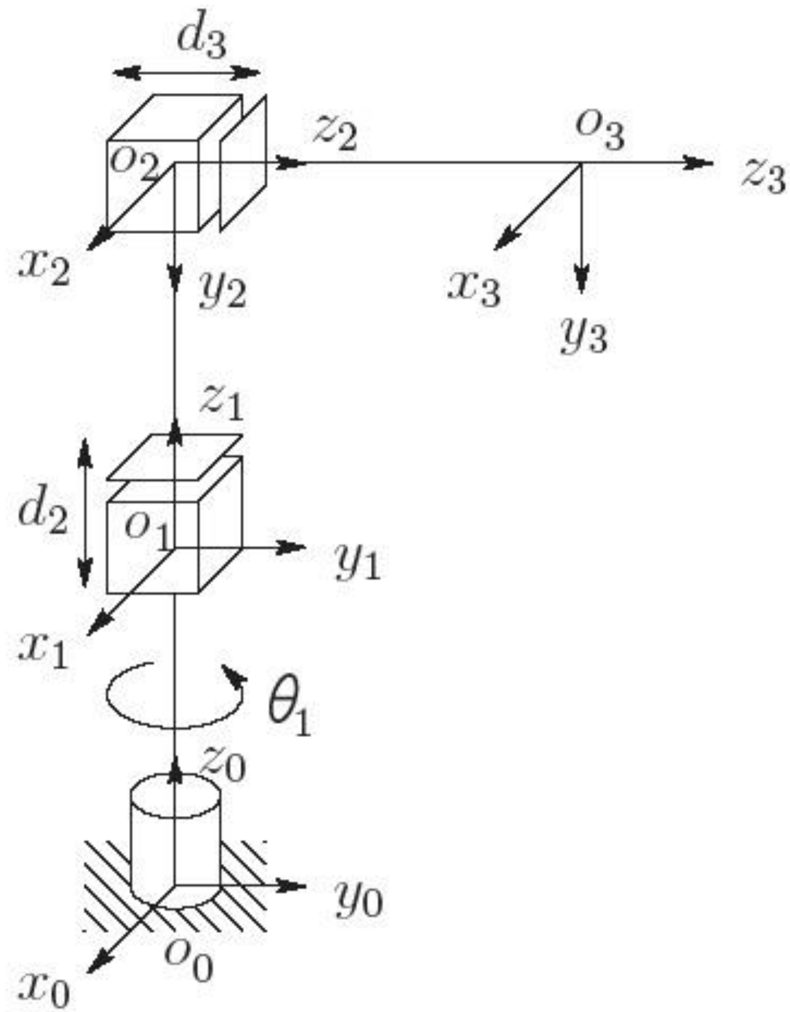


Figure 3.7: Three-link cylindrical manipulator.

Step 5: Find the DH parameters

- a_i : distance between z_{i-1} and z_i measured along x_i
- α_i : angle from z_{i-1} and z_i measured about x_i
- d_i : distance between o_{i-1} to the intersection of x_i and z_{i-1} measured along z_{i-1}
- θ_i : angle from x_{i-1} and x_i measured about z_{i-1}

Step 5: Find the DH parameters

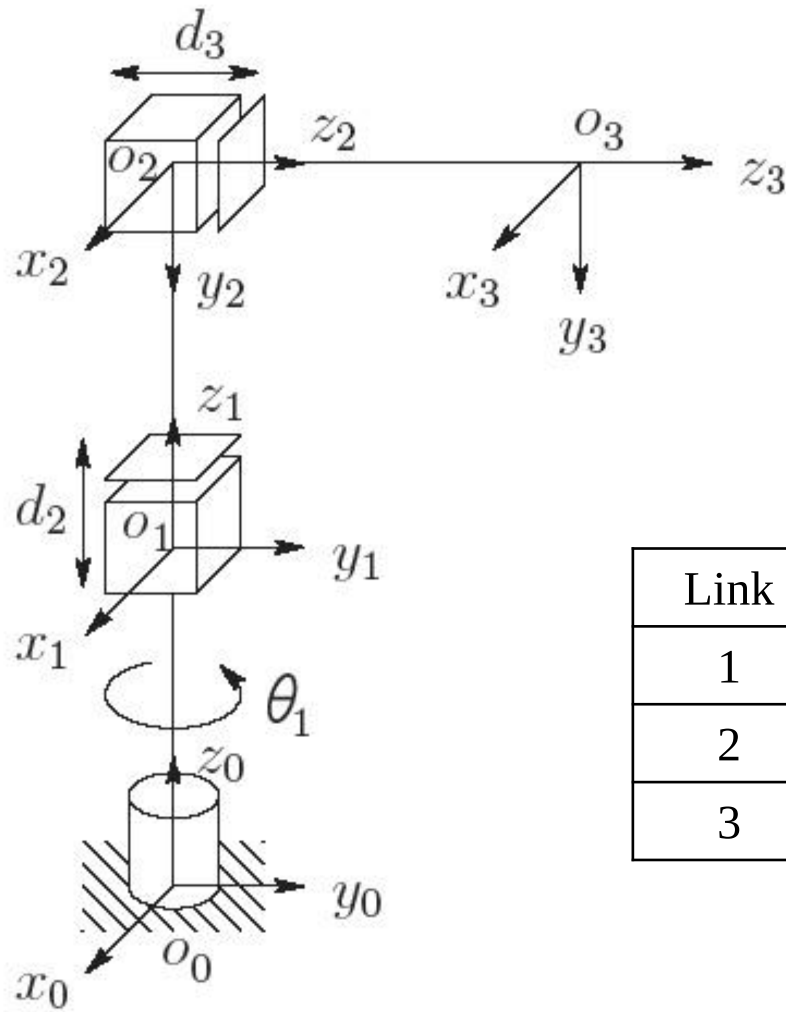


Figure 3.7: Three-link cylindrical manipulator.

Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1*
2	0	-90	d_2*	0
3	0	0	d_3*	0

* joint variable